On the Security of Some Code-Based (F)HE Approaches Using Reed-Muller Codes

WCC 2026

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DLR

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Goal: Perform arbitrary computations on encrypted data

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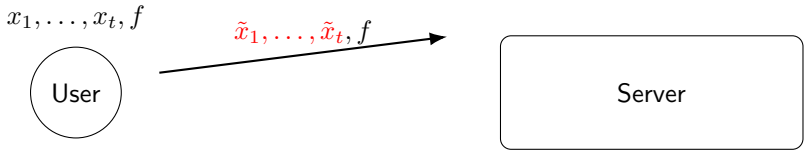

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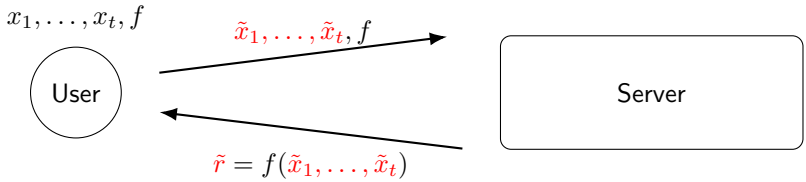

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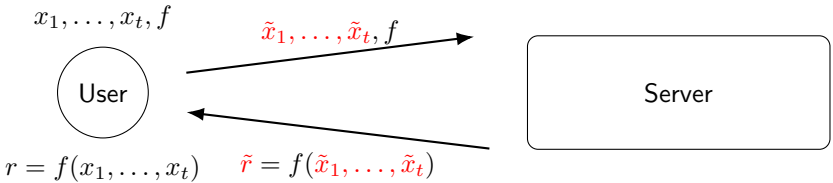

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- our contribution: identification of critical security vulnerabilities in the code-based scheme

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Notation:

- $\mathbb{F}_q[x_1, \ldots, x_t]_{\leq \rho}$ the polynomial ring in t variables with degree at most ρ
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For a set $S \subseteq \{1, \ldots, n\}$, $n = q^t$, the **punctured RM code** is

$$\mathcal{P}_S(\mathcal{C}) = \{(c_i)_{i \notin S} \mid (c_1, \ldots, c_n) \in \mathcal{C}\}.$$

Evaluation Codes

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Let $\mathcal{X}$ be a geometric object and $\mathcal{L} \subseteq \{f : \mathcal{X} \to \mathbb{F}_q\}$ a vector space of functions from $\mathcal{X}$ to $\mathbb{F}_q$. Let $P = (p_1, \ldots, p_n) \in \mathcal{X}^n$ be a tuple of distinct points. The **evaluation code** with respect to $\mathcal{L}$ and P is given by $\mathcal{C} = \{\mathrm{Eval}_P(f) : f \in \mathcal{L}\} \subseteq \mathbb{F}_q^n$.

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$\mathcal{C} = RM_q(m, \rho)$ and $\widehat{\mathcal{C}} = RM_q(m, \mu\rho)$

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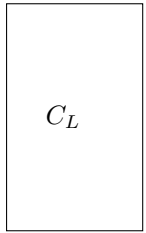

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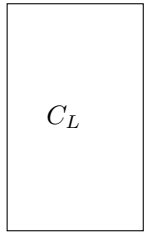

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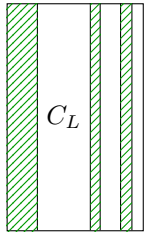

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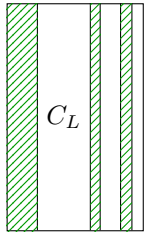

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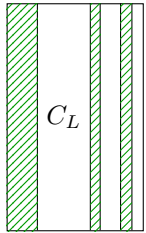

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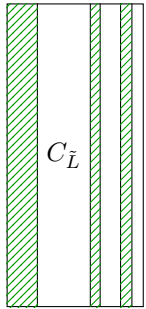

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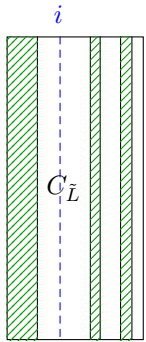

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$C_{\tilde{L}}[i]$

Partial Key Recovery

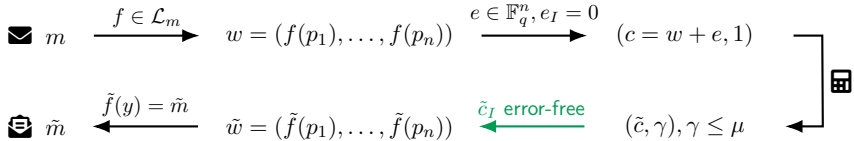
Lemma

If the number of ciphertexts $\tilde{L}$ is large enough, then it holds

- *if $i \notin I$, then* $\mathsf{rk}(C_{\tilde{L}}[i]) = \mathsf{rk}(C_{\tilde{L}}) - 1$ *with high probability,*
- *if $i \in I$, then* $\mathsf{rk}(C_{\tilde{L}}[i]) = \mathsf{rk}(C_{\tilde{L}})$ *with high probability.*

So, the error support can be recovered with a time complexity of $\mathcal{O}(n^4)$ with high probability.

Alternative Description



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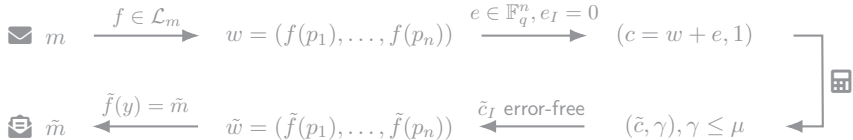

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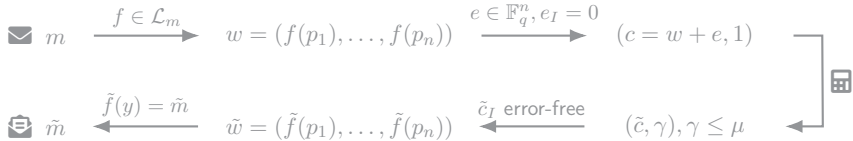

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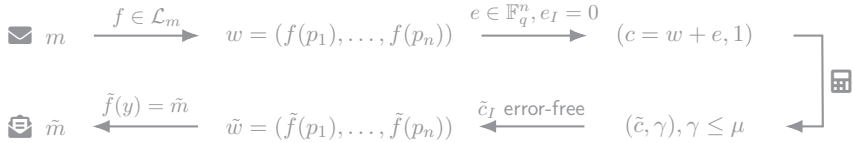

- $V_0 \subset \mathbb{F}_q^n$ subspace containing all encryptions of 0
- $c^* \in \mathbb{F}_q^n$ an arbitrary encryption of 1

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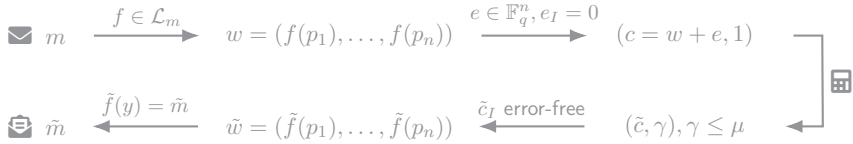

- $V_0 \subset \mathbb{F}_q^n$ subspace containing all encryptions of 0
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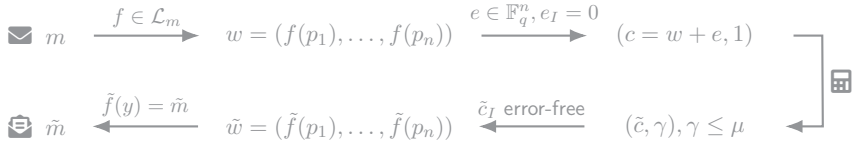

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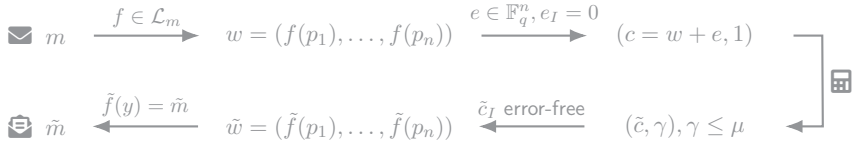

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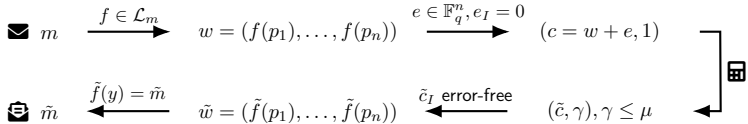
Message Recovery



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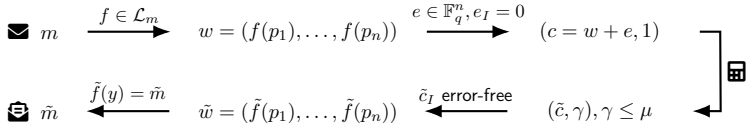

assume we know L' plaintext-ciphertext pairs m_i, c_i

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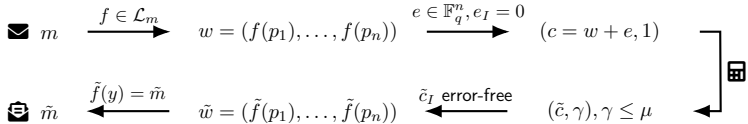

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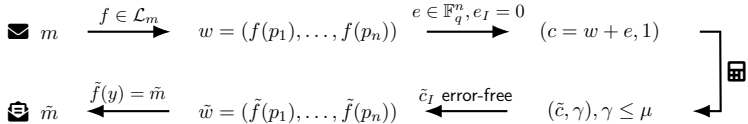

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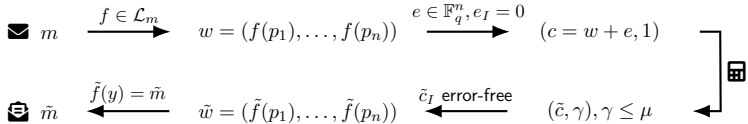

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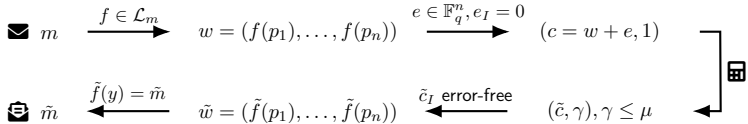

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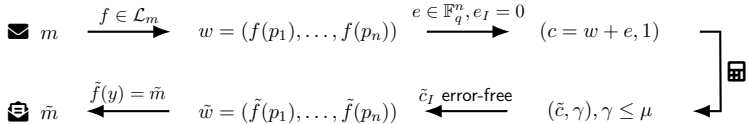

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Message Recovery

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4. We know there exists v_{key} such that $c_I \cdot v_{\text{key}}^\top = m$
5. Solve the linear system $\begin{pmatrix} (\tilde{c}_1)_I \\ \vdots \\ (\tilde{c}_\lambda)_I \end{pmatrix} v_{\text{key}}^\top = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$ to find v_{key}

Message Recovery

Theorem

Assume that we already recovered the set I.
If the number of known plaintext-ciphertext pairs L' is large enough, the messages can be recovered with a time complexity of $\mathcal{O}(n^3)$.

Parameters

	$s = 80$			$s = 128$		
	$\mu = 5$	$\mu = 10$	$\mu = 15$	$\mu = 5$	$\mu = 10$	$\mu = 15$
Scheme parameters						
Field order q	2^{115}	2^{119}	2^{122}	2^{165}	2^{169}	2^{172}
Number of evaluation points n	$60,176$	$448,017$	$1,475,597$	$114,189$	$862,336$	$2,853,838$
Code dimension k	165	165	165	455	286	286
Number of error-free positions $\lvert I\rvert$	$12,341$	$91,881$	$302,621$	$39,711$	$176,851$	$585,276$
Polynomial degree ρ	4	4	4	6	5	5
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Two additional FHE Approaches

Two FHE schemes based on binary RM codes by Challa and Gunta: [3]

3. R. Challa and V. Gunta. "Reed-Muller Code Based Symmetric Key Fully Homomorphic Encryption Scheme". In: *Information Systems Security*. Springer International Publishing, 2016
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Thank you for your attention!